Ex 1

$$P(s) = \frac{s-1}{s(s-3)} \qquad n=2 \quad m=1$$

$$C(s) = \frac{d_1 s + d_0}{\cancel{c_1} s + c_0} \qquad r = n-1 = 1$$

$$\text{den } W(s) = (s+c_0)\, s(s-3) + (d_1 s + d_0)(s-1)$$

$$= s^3 + (c_0 - 3 + d_1) s^2 + (d_0 - d_1 - 3c_0) s - d_0$$

$$\text{den } W_d(s) \overset{!}{=} s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$$

$$\Rightarrow \quad d_0 = -\alpha_0$$

$$\begin{bmatrix} 1 & 1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} c_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} \alpha_2 + 3 \\ \alpha_1 + \alpha_0 \end{bmatrix} \Rightarrow \begin{bmatrix} c_0 \\ d_1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & -\frac{1}{2} \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \alpha_2 + 3 \\ \alpha_1 + \alpha_0 \end{bmatrix}$$

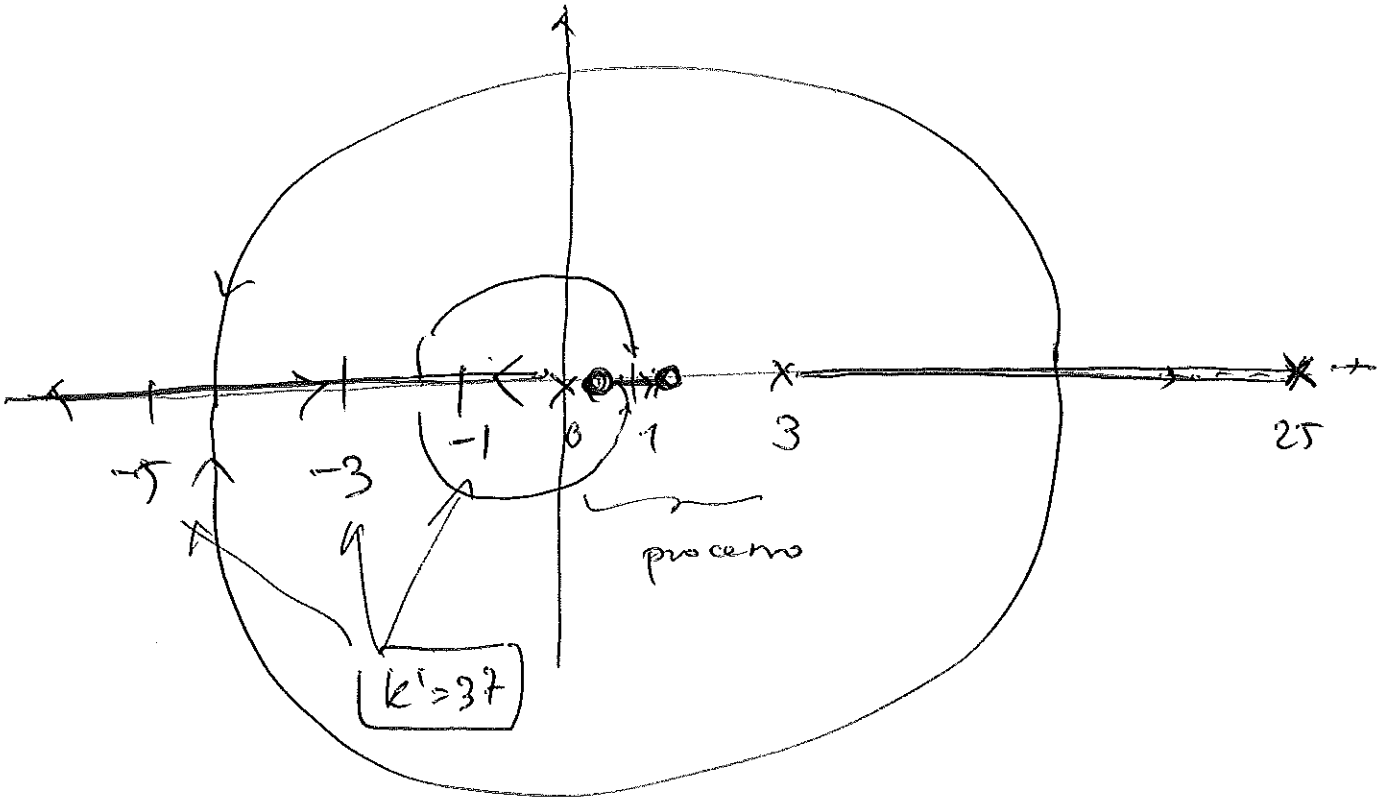
$$c_0 = -(\alpha_0 + \alpha_1 + \alpha_2 + 3)/2$$

$$d_1 = (\alpha_0 + \alpha_1 + 3\alpha_2 + 9)/2$$

e.g. $$\text{den } W_d(s) = (s+1)(s+3)(s+5) = s^3 + \underbrace{9}_{\alpha_2} s^2 + \underbrace{23}_{\alpha_1} s + \underbrace{15}_{\alpha_0}$$

$$\Rightarrow \quad C(s) = \frac{37 s - 15}{s - 25} = 37 \cdot \frac{s - 0.4054}{s - 25}$$

1. radici



per $k' = 37$ → radici in $-1, -3, -5$

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% Sintesi diretta per assegnazione dei poli
% ADL 20 May 2015

clc
clear all

% P(s) = (s-1)/s(s-3)

nP=[1 -1];
dP=[1 -3 0];

n=2;
m=1;
nm=n-m;

% polinomio desiderato (tre radici reali)
% den Wd(s)= (s+p1)(s+p2)(s+p3), con p1,p2,p3 >0

p1=1;
p2=3;
p3=5;
dWd=conv([1 p1+p2 p1*p2],[1 p3]);

al0=dWd(4);
al1=dWd(3);
al2=dWd(2);

% soluzione con controllore di dimensione r=n-1
% C(s) = (d1*s+d0)/(s+c0)

c0=-(al0+al1+al2+3)/2;
d0=-al0;
d1=(al0+al1+3*al2+9)/2;

disp('controllore risultante')
nC=[d1 d0]
dC=[1 c0]
pause;

disp('controllore nella forma poli-zeri e K''')
zeroscontroller=roots(nC)
polescontroller=roots(dC)
Kprimecontroller=d1
pause;

% root locus

nF=conv(nP,nC);
dF=conv(dP,dC);

rlocus(nF,dF)
pause;

% check closed-loop poles

dW=dF+[zeros(1,nm) nF];
desiredroots=roots(dWd)
obtainedroots=roots(dW)
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Ex 2

$$P(s) = \frac{s-1}{s(s-3)(s+2)}$$

$n = 3$ $\quad n - m = 2$

$m = 1$

$r = n - 1 = 2$

$$C(s) = \frac{d_2 s^2 + d_1 s + d_0}{s^2 + c_1 s + c_0}$$

5 parametri $= 2n - 1$

$= 2r + 1$

$$\text{den } W(s) = (s^2 + c_1 s + c_0)\, s\, (s-3)(s+2) + (d_2 s^2 + d_1 s + d_0)(s-1)$$

$$\begin{aligned} = \; & s^5 \cdot 1 \\ & + s^4 \cdot (c_1 - 1) \\ & + s^3 \cdot (c_0 - c_1 - 6 + d_2) \\ & + s^2 \cdot (-c_0 - 6c_1 - d_2 + d_1) \\ & + s \cdot (-6c_0 - d_1 + d_0) \\ & - d_0 \end{aligned}$$

$$\text{den } W_{des}(s) = s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$$

$$\Rightarrow \quad \begin{aligned} d_0 &= -\alpha_0 \\ c_1 &= \alpha_4 + 1 \end{aligned} \qquad + \quad \underbrace{\begin{bmatrix} -6 & -1 & \cancel{-\alpha_0} \\ -1 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}}_{A} \begin{bmatrix} c_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} + \begin{bmatrix} +\alpha_0 \\ 6(\alpha_4 + 1) \\ \alpha_4 + 7 \end{bmatrix}$$

$$\det A = -\frac{1}{6}\begin{bmatrix} +1 & +1 & +1 \\ 0 & -6 & -6 \\ +1 & +1 & +7 \end{bmatrix}$$

$$\begin{bmatrix} c_0 \\ d_1 \\ d_2 \end{bmatrix} = \begin{matrix} -(\alpha_1+\alpha_2+\alpha_3+\alpha_0+7\alpha_4+13)/6 \\ \alpha_2+\alpha_3+7\alpha_4+13 \\ (\alpha_1+\alpha_2+7\alpha_3+\alpha_0+13\alpha_4+55)/6 \end{matrix}$$

$$d_0 = -\alpha_0$$

$$c_1 = \alpha_4+1$$

Esempi

i) tutti i poli in $s=-3$ $\therefore$ den $W_{des}(s) = (s+3)^5$

$$= s^5 + \underset{\substack{\uparrow \\ \alpha_4}}{15}s^4 + 90s^3 + 270s^2 + 405s + \underset{\substack{\uparrow \\ \alpha_0}}{243}$$

$$C(s) = \frac{299{,}\bar{6}s^2 + 478s - 243}{s^2+16s-187.\bar{6}} \sim \frac{299{,}\bar{6}\,(s-2{,}005)(s-0.4054)}{(s+23.864)(s-7.864)}$$

N.B. Cancellazione zero f.m. controllore / polo stabile proc.

ii) tutti i poli in $s=-2$: den $W_{des}(s) = (s+2)^5$ qui

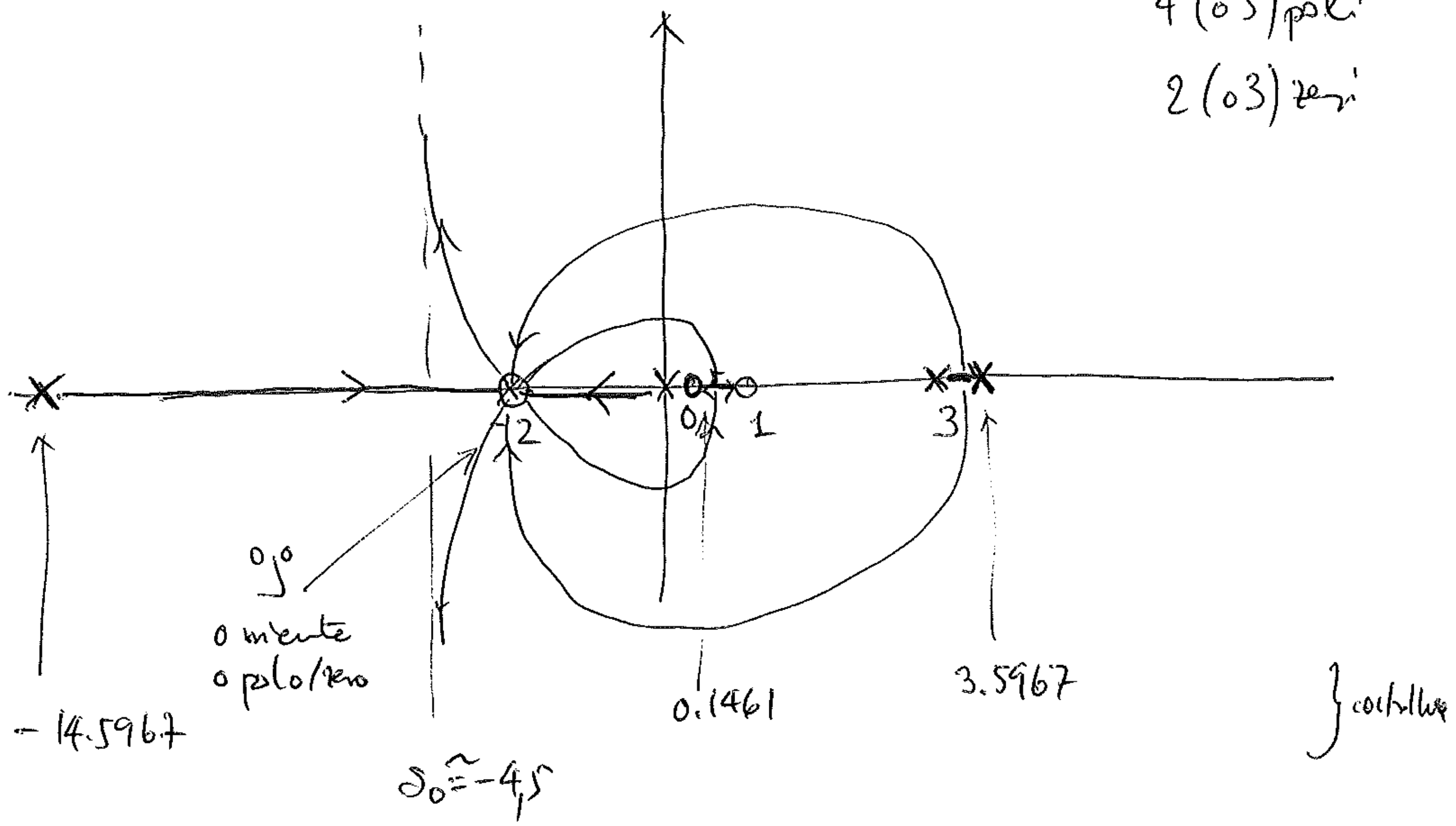
$$= s^5 + 10s^4 + 40s^3 + 80s^2 + 80s + 32$$

$$C(s) = \frac{109.5s^2 + 203s - 32}{s^2+11s-52.5s} = 109.5\,\frac{\boxed{s+2}\,(s-0.1461)}{(s+14.5967)(s-3{,}5967)}$$

4

1. radici (caso ii))

4 (o 5) poli
2 (o 3) zeri

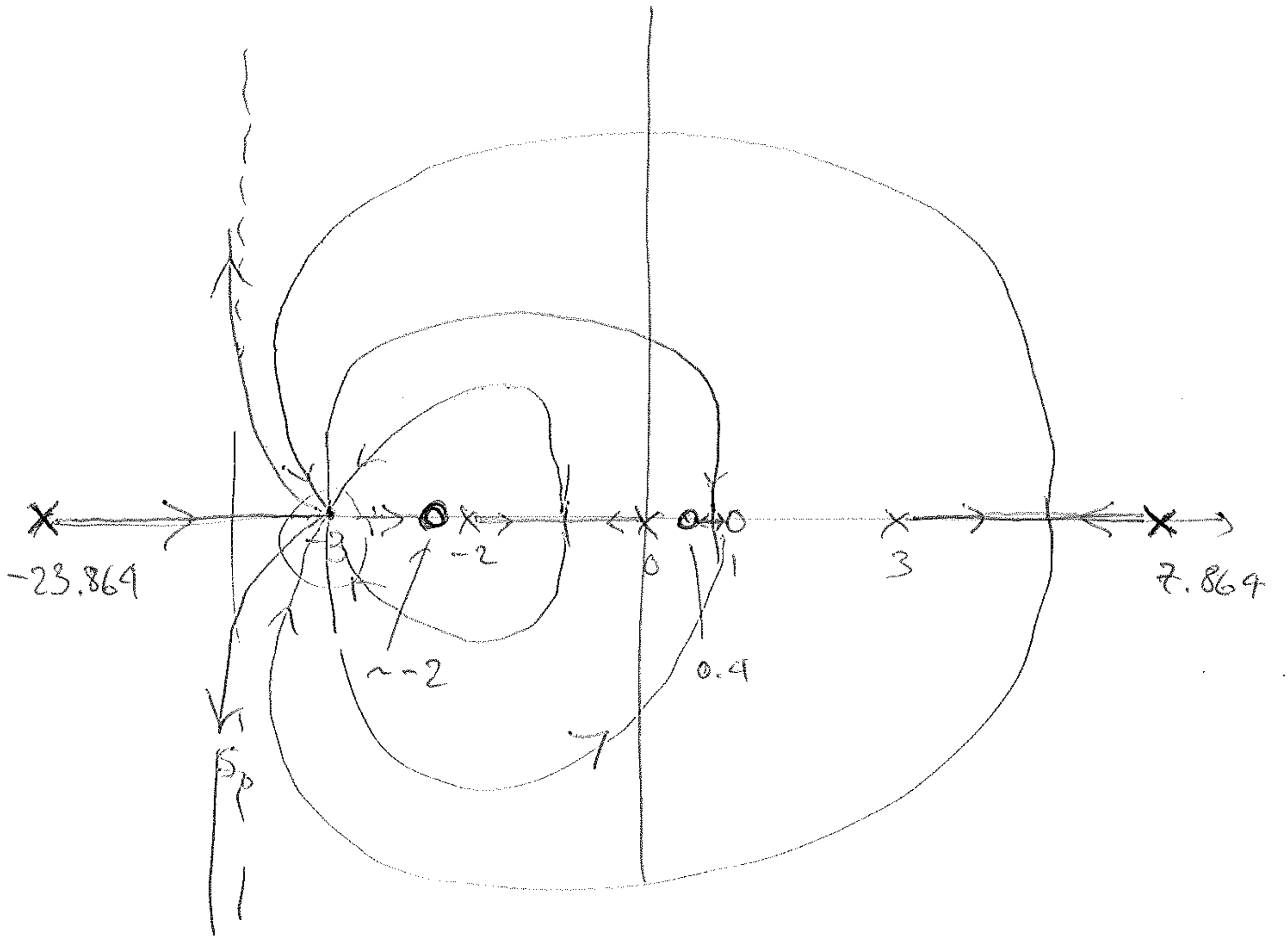


con $K' = 109.5$ (l. positivo) tutti i 4 poli in -2

$$s_0 = \frac{\sum p_i - \sum z_i}{n-m} = \frac{(+3 - \cancel{2}) + 3.59 - 14.59) - (1 - \cancel{2} + 0.146)}{2}$$

$$\simeq -4.5$$

l. radici caso i)



5 radici in -3 per K' = 299,6̄

```
% Sintesi diretta per assegnazione dei poli
% ADL 20 May 2015

clc
clear all

% P(s) = (s-1)/s(s-3)(s+2)

nP=[1 -1];
dP=[conv([1 -3],[1 2]) 0];

n=3;
m=1;
nm=n-m;

% polinomio desiderato (poli reali coincidenti)
% den Wd(s)= (s+a)^5, con a >0

a=2; %a=3 oppure a=2 (cancellazione polo processo/zero controllore ad anello aperto)
dWd=conv(conv(conv(conv([1 a],[1 a]),[1 a]),[1 a]),[1 a]);

al0=dWd(6);
al1=dWd(5);
al2=dWd(4);
al3=dWd(3);
al4=dWd(2);

% soluzione con controllore di dimension r=n-1
% C(s) = (d2*s^2+d1*s+d0)/(s^2+ c1*s+ c0)

c0=-(al1+al2+al3+al0+7*al4+13)/6;
c1=al4+1;
d0=-al0;
d1=al2+al3+7*al4+13;
d2=(al1+al2+7*al3+al0+13*al4+55)/6;

disp('controllore risultante')
nC=[d2 d1 d0]
dC=[1 c1 c0]
pause;

disp('controllore nella forma poli-zeri e K''')
zeroscontroller=roots(nC)
polescontroller=roots(dC)
Kprimecontroller=d2
pause;

% root locus

nF=conv(nP,nC);
dF=conv(dP,dC);

rlocus(nF,dF)
pause;

% check closed-loop poles

dW=dF+[zeros(1,nm) nF];
desiredroots=roots(dWd)
obtainedroots=roots(dW)
```